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General Relativity and General Lorentz-covariance

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Abstract

The principle of general relativity means the principle of general *Lorentz*-covariance of the physical equations in the language of tetrads and metrical spinors. A general *Lorentz*-covariant calculus and the general *Lorentz*-covariant generalisations of the Ricci calculus and of the spinor calculus are given. The general *Lorentz*-covariant representation implies the *Einstein* principle of space-time covariance and allows the geometrisation of gravitational fields according to *Einstein's* principle of equivalence.

1. *The Meaning of General Lorentz-Covariant Derivatives*

To understand the principle of general relativity it is necessary to distinguish between systems of coordinates $\{x^i\}$ and systems of reference Σ: the former are quite mathematical and describe mathematical relations. The independence of physical quantities of the choice of the coordinate system is a logical necessity, but implies no physical consequences. On the other hand systems of reference have physical reality; they correspond to an arrangement of measurements, which determine the physical quantities.

In the simplest case such a system is realised by three measuring rods and one normal clock. To every event of the space-time V_4 three rods and a clock are attached (Treder, 1966).

A system of reference Σ is represented mathematically by a field of four vectors $h^A{}_i$, which can be assumed to be orthonormalised:

$$g_{ik} = h^A{}_i h^B{}_k \eta_{AB}, \qquad \eta_{AB} = h_A{}^i h_B{}^k g_{ik} \tag{1.1}$$

Here, g_{ik} means the metric tensor of the space-time,

$$\eta_{AB} = \mathrm{diag}(-1, -1, -1, +1)$$

the *Minkowski* tensor; the Latin minuscules are tensorial indices in the space-time and the Latin capitals denumerate the vectors, both indices run from 1 to 4. The $h^A{}_i$ themselves are functions of the space-time-coordinates x^1

$$\frac{\partial h^A}{\partial x^l} \underset{\text{def}}{=} h^A{}_{i,l} \neq 0$$

Equation (1.1) is the condition for compatibility of the system of reference with a space-time of given metric g_{ik}. A coordinate transformation from the *Einstein* group

$$x'^{l} = x'^{l}(x^{k}), \qquad h'^{A}{}_{l} = \frac{\partial x^{k}}{\partial x'^{l}} h^{A}{}_{k} \tag{1.2}$$

transforms the tetrad vectors corresponding to the metric

$$g'_{mm} = \frac{\partial x^{k}}{\partial x'^{m}} h^{A}{}_{k} \frac{\partial x^{l}}{\partial x'^{m}} h_{Al} = \frac{\partial x^{k}}{\partial x'^{m}} \frac{\partial x^{l}}{\partial x'^{m}} g_{kl} \tag{1.3}$$

Equation (1.1) assigns universal *Minkowski* tangent space M_4 to the space time V_4. This M_4 represents the manifold $V_4{}^{+}$ dual to V_4. The transformation matrix $h^{A}{}_{l}$, connecting V_4 and $V_4{}^{+}$, is anholonomic in general with the *Einstein* object of anholonomy (Einstein, 1928; Schouten, 1953):

$$\underset{\gamma}{\Delta}{}^{i}{}_{kl} = \tfrac{1}{2} h_{A}{}^{i}(h^{A}{}_{k,l} - h^{A}{}_{l,k}) \tag{1.4}$$

the $h^{A}{}_{i}$ being a system of reference compatible with the given metric g_{ik}. Also, the tetrad $\bar{h}^{A}{}_{k}$ (*Lorentz*-rotated in the *Minkowski* space $V_4{}^{+}$) represents a compatible system of reference:

$$\bar{h}^{B}{}_{i} = \omega^{B}{}_{A} h^{A}{}_{i} \tag{1.5}$$

with

$$\omega_{A}{}^{C} \omega_{BC} = \eta_{AB} \tag{1.6}$$

The principle of general relativity now requires the equivalence of all systems of reference compatible with the given metric structure g_{ik} of the space-time.

This supposition is realised by the geometric objects of the space-time V_4 iff these depend on the *Lorentz* invariant combination (1.1) of the tetrads and its derivatives only (see later).

The measured values ϕ^{T} of physical quantities are invariant with regard to choice of coordinate system, they have to be pure functions of the point and therefore scalars in the space-time:

$$\phi'^{T} = \phi^{T}(x^{i}(x'^{l})) = \phi^{T}(x^{i}) \tag{1.7}$$

Corresponding to the principle of relativity, the relations between measured values of physical quantities also have to be independent of the system of reference chosen. Especially, we have to require that $\bar{\phi}^{T} = 0$ iff $\phi^{T} = 0$. From this it follows that the matrix has to behave covariantly with respect to the *Lorentz* transformations (1.6), i.e. the matrix of the measured values ϕ^{T} has to be a *Lorentz* tensor of any degree:

$$\bar{\phi}^{A_1 \cdots}{}_{B_1 \ldots} = \omega^{A_1}{}_{C_1} \ldots \omega_{B_1}{}^{D_1 \cdots} \phi^{C_1 \cdots}{}_{D_1 \ldots} \tag{1.8}$$

From (1.7) and (1.8) it follows by (1.1) that a space-time tensor of the same degree can be coordinated unambiguously to every *Lorentz* tensor of measured values of physical quantities:

$$\phi^{i_1\cdots}{}_{k_1\ldots} = h_{A_1}{}^{i_1\cdots}\, h^{B_1}{}_{k_1\ldots}\, \phi^{A_1\cdots}{}_{B_1\ldots} \tag{1.9}$$

These space-time tensors for their part are *Lorentz* invariant. *Lorentz* tensors $\phi^{A\cdots}{}_{B\ldots}$ and space-time tensors $\phi^{i\cdots}{}_{k\ldots}$ are dual quantities.

The ordinary differential $d\phi^{i\cdots}{}_{k\ldots}$ of a space-time tensor itself is not a tensor, for instance:

$$d\phi'^i = \phi'^i_{,l}\, dx^l = \left(\frac{\partial x'^i}{\partial x^r}\phi^r{}_{,l} + \frac{\partial z\, x'^i}{\partial x^r\, \partial x^l}\phi^r\right) dx^l \tag{1.10a}$$

The differential becomes a tensor by addition of a compensating quantity

$$\Gamma^i_{kl}\,\phi^k\, dx^l \tag{1.10b}$$

where the affinity Γ^i_{kl} satisfies the transformation law

$$\Gamma'^i{}_{kl} = \frac{\partial x'^i}{\partial x^m}\frac{\partial x^r}{\partial x'^k}\frac{\partial x^P}{\partial x'^l}\Gamma^m_{rp} + \frac{\partial x'^i}{\partial x^m}\frac{\partial^{2m}_{x}}{\partial x'^k\,\partial x'^l} \tag{1.10c}$$

From the g_{ik} and their derivatives alone, the *Christoffel* symbols only can be constructed as affinity:

$$\Gamma^i_{kl} = \left[{i \atop kl}\right] = \tfrac{1}{2} g^{ir}[-g_{kl,r} + g_{lr,k} + g_{rk,l}] \tag{1.10d}$$

They are *Lorentz* invariant. Likewise, the ordinary differential

$$d\phi^A = \phi^A{}_{,l}\, dx^l = \phi^A{}_{,l}\, h^l{}_B\, dx^B \underset{\text{def}}{=} \phi^A{}_{,B}\, dx^B \tag{1.11a}$$

of a *Lorentz* vector ϕ^A is not a *Lorentz* tensor:

$$d\overline{\phi^A} = (\omega^A{}_B\,\phi^B)_{,l}\, dx^l \tag{1.11b}$$

$$= (\omega^A{}_B\,\phi^B{}_{,l} + \omega^A{}_{B,l}\,\phi^B)\, dx^l$$

A *Lorentz* covariant derivative has to be constructed using a compensating *Lorentz* affinity

$$L^A{}_{Bl}\,\phi^B\, dx^l = L^A{}_{BC}\,\phi^B\, dx^C \tag{1.11c}$$

with the transformation law

$$\bar{L}^A{}_{Bl} = \omega^A{}_C\,\omega^D{}_B\, L^C{}_{Dl} + \omega^A{}_D\,\omega_B{}^D{}_{,l} \tag{1.11d}$$

The only *Lorentz* affinity which can be constructed by the tetrads and their derivatives alone reads†

$$L^A{}_{Bl} = -\gamma^A{}_{Bl} = +h^A{}_i\, h^i_{B:l} = -h_B{}^i\, h^A{}_{i:l} \tag{1.12a}$$

† Starting in a flat V_4 from a special relativistic inertial frame, which reads $h^A{}_i = \delta^A{}_i$ in cartesian coordinates, and performing general *Lorentz* transformation $h^A{}_i = \omega^A{}_B\delta^B{}_i$ we find

$$\gamma^A{}_{Bl} = -\omega^A{}_C\,\omega_B{}^C{}_{,l}$$

Indeed these quantities realise the transformations law

$$\bar{\gamma}^{A}{}_{Bl} - \omega^{A}{}_{C}\,\omega_{B}{}^{D}\,\gamma^{C}{}_{Dl} = \omega^{A}{}_{D}\,\omega_{B}{}^{D}{}_{,l} = +\omega^{A}{}_{D,l}\,\omega_{B}{}^{D} \tag{1.12b}$$

The quantities (1.12a) are the *Ricci* rotation coefficients of the tetrad field $h^{A}{}_{i}$. Their space-time components (Eisenhart, 1927)

$$\gamma^{i}{}_{kl} = h_{A}{}^{i}\,h^{B}{}_{k}\,\gamma^{A}{}_{Bl} = h_{A}{}^{i}\,h^{A}{}_{k:l} \tag{1.12c}$$

$$(\phi^{i}{}_{:l} = \phi^{i}{}_{;l} + \phi^{r}\,\Gamma^{i}{}_{rl})$$

are not legitimate general relativistic quantities, they actually are space-time tensors, but not *Lorentz* scalars. In that respect they are analogous to the *Christoffel* affinity $\Gamma^{i}{}_{kl}$, which is *Lorentz* scalar, but not a space-time tensor.

Using the *Lorentz* affinity (1.12a) we define a *Lorentz* covariant derivative with the *Leibniz* rule, for instance the derivative of a mixed *Lorentz* tensor $\phi^{A}{}_{D}$ by

$$\phi^{A}{}_{B//C} = \phi^{A}{}_{B//l}\,h_{C}{}^{l} = \phi^{A}{}_{B,C} - \gamma^{A}{}_{DC}\,\phi^{D}{}_{B} + \gamma^{D}{}_{BC}\,\phi^{A}{}_{D} \tag{1.13}$$

For a mixed space-time and *Lorentz* tensor, e.g. $\phi^{A}{}_{i}$, we define the general covariant (*Lorentz* covariant and coordinate covariant) derivative using the *Ricci* rotation coefficients and the *Christoffel* affinity by

$$\phi^{A}{}_{i///l} = \phi^{A}{}_{i,l} - \phi^{B}{}_{i}\,\gamma^{A}{}_{Bl} - \phi^{A}{}_{r}\,\Gamma^{r}{}_{il} \tag{1.14a}$$

This general covariant derivative specialises to the coordinate covariant derivative

$$\phi^{i}{}_{///l} = \phi^{i}{}_{,l} + \Gamma^{i}{}_{rl}\,\phi^{r} = \phi^{i}{}_{:l} \tag{1.14b}$$

for pure space-time quantities and to the *Lorentz* covariant derivative

$$\phi^{A}{}_{///l} = \phi^{A}{}_{,l} - \gamma^{A}{}_{Bl}\,\phi^{B} = \phi^{A}{}_{//l} \tag{1.14c}$$

for pure *Lorentz* tensors.

Besides the known indentities

$$\delta^{i}{}_{k;l} = \delta^{i}{}_{k,l} = 0 \tag{1.15a}$$

$$\delta^{A}{}_{B//l} = \delta^{A}{}_{B,l} = 0 \tag{1.15b}$$

$$g_{ik///l} = g_{ik:l} = 0 \tag{1.15c}$$

$$\eta_{AB///l} = \eta_{AB//l} = \gamma_{ABl} + \gamma_{BAl} = 0 \tag{1.15d}$$

we find also the covariant constance of the tetrads

$$\begin{aligned} h^{A}{}_{i///l} &= h^{A}{}_{i,l} - h^{B}{}_{i}\,\gamma^{A}{}_{Bl} - h^{A}{}_{r}\,\Gamma^{r}{}_{il} \\ &= h^{A}{}_{i,l} - h^{A}{}_{i;l} - h^{A}{}_{r}\,\Gamma^{r}{}_{il} = 0 \end{aligned} \tag{1.16}$$

Indeed

$$\gamma^{i}{}_{kl} + \Gamma^{i}{}_{kl} = \Delta^{i}{}_{kl} = h_{A}{}^{i}\,h^{A}{}_{k,l} \tag{1.17}$$

is the *Einstein* affinity with integrable parallelism (Einstein, 1928).

Because of (1.1) and (1.16) there exists a unique correspondence (duality) between the *Lorentz* covariant and the coordinate covariant representation of tensorial quantities. For instance, it yields

$$\phi^A{}_{//C} = (h^A{}_i\,\phi^i)_{//l}\,h^l{}_C = \phi^i{}_{;l}\,h^A{}_i\,h_C{}^l \tag{1.18a}$$

(Eisenhart, 1927) and

$$\phi^A{}_{//CD} = \phi^i{}_{;kl}\,h^A{}_i\,h_C{}^k\,h_D{}^l \tag{1.18b}$$

Lorentz covariant derivatives of *Lorentz* tensors can be replaced by coordinate covariant derivatives of space-time tensors, and vice versa. Both types of covariant derivatives are dual to each other.

Especially, the curvature tensor of the *Lorentz* affinity equals the *Riemann* curvature tensor. It yields

$$\begin{aligned}\phi^A{}_{//BC} - \phi^A{}_{//CB} &= (\phi^i{}_{;kl} - \phi^i{}_{;lk})\,h^A{}_i\,h_B{}^k\,h_C{}^l \\ &= -R^i{}_{rkl}\,\phi^r\,h^A{}_i\,h_B{}^k\,h_C{}^l = -\phi^D\,R^A{}_{DBC}\end{aligned} \tag{1.19}$$

The theorem of inertia in special relativity

$$u^l{}_{,k}\,u^k = 0$$

can be expressed by four scalar equations in a *Lorentz* covariant way

$$u^A{}_{//C}\,u^C = (u^A{}_{,i} - \gamma^A{}_{Bi}\,u^B)\,h^i{}_C\,u^C = 0 \tag{1.20a}$$

and therefore it leads to the geodesic equation in a *Riemann* space V_4:

$$u^i{}_{;k}\,u^k\,h_i{}^A = u^A{}_{//C}\,u^C = 0 \tag{1.20b}$$

The requirement of covariance against any point-dependent *Lorentz* rotation is of decisive importance in the derivation of the geodesic equation. Only the covariance against rigid *Lorentz* rotations required ($\omega^A{}_{B,l} = 0$), the ordinary differential

$$u^i{}_{;k}\,u^k\,h_i{}^A = u^A{}_{//C}\,u^C = 0$$

had not to be completed by a *Lorentz* affinity:

$$\phi^A{}_{,l}\,dx^l = (\phi^i{}_{,l}\,h^A{}_i + h^A{}_{i,l}\,\phi^i)\,dx^l \tag{1.21}$$

(1.21) lead to the dual coordinate covariant derivative

$$d\phi^A\,h_A{}^k = (\phi^k{}_{,i} + h_A{}^k\,h^A{}_{i,l}\,\phi^i)\,dx^l \tag{1.22}$$

In these differentials the coordinate affinity equals the integrable *Einstein* connection $\Delta^i{}_{kl}$. Then the tensorial transport in the V_4 is integrable, there is no affine curvature and therefore no gravitational action. Accordingly the principle of general relativity implies the *Einstein* principle of space-time covariance of the physical equations formulated in space-time by the requirement of *Lorentz* covariance of the relations between measured values of physical quantities and enables the geometrisation of the gravitational field.

Under the assumption that the structure of the space-time is given only by the metric g_{ik} and that any physical quantity is entirely given by *Lorentz* tensors, it is possible to write the physical equations unambiguously by space-time tensors, independent of any system of reference, and the principle of general relativity reads in this case: 'The basic physical equations can be formulated in the V_4 without referring to any system of reference (*Einstein* principle of general relativity).'

For the relations between the measured values following from these basic equations in V_4 to be independent of the choosen coordinate system (and therefore not to forbid interpretation), these equations in space-time have to be coordinate covariant (*Einstein* principle of covariance). The *Einstein* form of the principle of general relativity represents the dual counterpart of the requirement of *Lorentz* covariance.

2. *General Lorentz-Covariant Calculus and Spinor-Calculus*

There are two equivalent representations (dual to each other) for tensor fields, realizing the principle of general relativity (Treder, 1966):

(1) the *Lorentz* covariant representation using coordinate invariant quantities $\phi^{A\cdots}{}_{B\ldots}$, and

(2) the coordinate covariant representation using *Lorentz* invariant quantities $\phi^{i\cdots}{}_{k\ldots}$.

The equivalence of both representations follows from the unambiguity of the assignment

$$\phi^{A\cdots}{}_{B\ldots} = h^A{}_i \cdots h_B{}^k \cdots \phi^{i\cdots}{}_{k\ldots} \tag{2.1a}$$

and

$$\phi^{i\cdots}{}_{k\ldots} = h_A{}^i \cdots h^B{}_{k\ldots} \phi^{A\cdots}{}_{B\ldots} \tag{2.1b}$$

Such an unambiguous coordination becomes impossible as soon as spinor quantities are introduced. The introduction of spinor quantities implies at first nothing further for tensor fields than the replacement of the single-valued tensor representation of the *Lorentz* group by the double-valued unimodular representation. At this metric spinors are assigned to the tetrads of the systems of reference by

$$\sigma^{l\mu\dot{\nu}} = h_A{}^l \sigma^{A\mu\dot{\nu}} \tag{2.2}$$

In (2.2) the $\sigma^{A\mu\dot{\nu}}$ are the constant *Pauli* spin matrices. The Greek indices are spinor indices and run from 1 to 2.

The orthogonality condition (1.1) now reads

$$\gamma_{\alpha\beta}\gamma_{\dot{\mu}\dot{\nu}} = \sigma^k{}_{\alpha\mu}\sigma^l{}_{\beta\dot{\nu}} g_{kl} = \sigma^A{}_{\alpha\dot{\mu}}\sigma^B{}_{\beta\dot{\nu}}\eta_{AB} \tag{2.3a}$$

and

$$g_{kl} = \sigma_k{}^{\alpha\dot{\mu}}\sigma_l{}^{\beta\dot{\nu}}\gamma_{\alpha\beta}\gamma_{\dot{\mu}\dot{\nu}} = h^A{}_k h^B{}_l \sigma_A{}^{\alpha\mu}\sigma_B{}^{\beta\dot{\nu}}\gamma_{\alpha\beta}\gamma_{\dot{\mu}\dot{\nu}} \tag{2.3b}$$

Here the quantities

$$\gamma_{\alpha\beta} = -\gamma_{\beta\alpha} = \gamma_{\dot{\alpha}\dot{\beta}} = -\gamma_{\dot{\alpha}\dot{\beta}} \qquad \text{with} \qquad |\gamma_{\alpha\beta}| = 1, \qquad \gamma_{\alpha\beta}\gamma^{\epsilon\beta} = \delta_\alpha{}^\epsilon \tag{2.4a}$$

are the metric tensors of the spinor space and by (2.3) equal to

$$\gamma_{12} = -\gamma_{21} = -\gamma^{12} = \gamma^{21} = 1 \tag{2.4b}$$

The representation (2.3) is invariant with respect to unimodular transformations in spinor space S_2 resp. $S_2{}^*$, i.e. transformations

$$\bar{\sigma}_i{}^{\alpha\dot{\mu}} = \alpha^{\dot{\mu}}{}_{\dot{\nu}}\, \alpha^{\alpha}{}_{\beta}\, \sigma_i{}^{\beta\dot{\nu}} \tag{2.5}$$

of the metric spinors, where the transformation matrices $\alpha^{\mu}{}_{\nu} = \alpha^{\mu}{}_{\nu}(x^l)$ realize the condition

$$|\alpha^{\dot{\mu}}{}_{\dot{\nu}}| = |\alpha^{\mu}{}_{\nu}| = 1 \tag{2.6}$$

The measured values being referred to the spinor spaces S_2, $S_2{}^*$, the principle of general relativity implies that all measured values transform as spinors—corresponding to the unimodular invariance of (2.3)

$$\bar{\psi}^{\nu} = \alpha^{\nu}{}_{\mu}\, \psi^{\mu}, \qquad \bar{\psi}^{\dot{\nu}} = \alpha^{\dot{\nu}}{}_{\dot{\mu}}\, \psi^{\dot{\mu}} \tag{2.7a}$$

Corresponding to (2.2) this condition for spinors of degree $2n$, like

$$\psi_{\mu\dot{\nu}}, \qquad \psi_{\mu\dot{\nu}\alpha\dot{\beta}}, \text{ etc.}$$

is equivalent to the condition that the measured values are *Lorentz* tensors:

$$\phi_{\mu\dot{\nu}} = \sigma_{A\mu\dot{\nu}}\, \phi^A = \sigma_{i\mu\dot{\nu}}\, h_A{}^i\, \phi^A = \sigma_{i\mu\dot{\nu}}\, \phi^i \tag{2.7b}$$

ψ_ν being a spinor, the ordinary derivative

$$d\psi_\nu = \psi_{\nu,l}\, dx^l \tag{2.8a}$$

is no spinor, for we have

$$\bar{\psi}_{\nu,l} = (\alpha_\nu{}^\mu\, \psi_\mu)_{,l} = \alpha_\nu{}^\mu\, \psi_{\mu,l} + \alpha_\nu{}^\mu{}_{,l}\, \psi_\mu \tag{2.8b}$$

By the introduction of some compensating spinor affinity $\Lambda^\alpha{}_{\beta l}$ with the transformation law

$$\bar{\Lambda}^\alpha{}_{\beta l} = \alpha_\beta{}^\mu\, \alpha^\alpha{}_\nu\, \Lambda^\nu{}_{\mu l} + \alpha^k{}_\lambda\, \alpha^\lambda{}_{\beta,l} \tag{2.8c}$$

the corresponding *Lorentz* covariant spinor derivative (Iwanenko, 1965; and *Elementary Particles and Compensatory Fields*. Moskwa, 1964)

$$\psi_{\nu//l}\, dx^l = (\psi_{r,l} - \Lambda^\mu{}_{\nu l}\, \psi_\mu)\, dx^l \tag{2.9}$$

is constructed. The spinor affinity has to be a space-time tensor for the spinor derivative (2.9) to be coordinate covariant. The only affinity of this kind which can be constructed from the metric spinors and their derivatives alone is the affinity of Infeld and van der Waerden (1933):

$$\Lambda^\alpha{}_{\beta l} = \tfrac{1}{2}\sigma^{i\alpha\dot{\nu}}\, \sigma_{i\beta\dot{\nu};l} = \tfrac{1}{2}\sigma^{i\alpha}{}_{\dot{\nu};l}\, \sigma_{i\beta}{}^{\dot{\nu}} \tag{2.10}$$

Again we are able to define general covariant derivatives for mixed tensor and spinor quantities, for example

$$\phi^k{}_{\nu///l} = \phi^k{}_{\nu,l} + \Gamma^k{}_{rl}\, \phi^r{}_\nu - \Lambda^\alpha{}_{\nu l}\, \phi^k{}_\alpha \tag{2.11}$$

By (2.11) we obtain the known general covariant constance of the metric spinors

$$\sigma^{k\mu\dot{\nu}}{}_{///l} = \sigma^{k\mu\dot{\nu}}{}_{,l} + \sigma^{r\mu\dot{\nu}}\,\Gamma^{k}{}_{rl} + \sigma^{k\alpha\dot{\nu}}\,\bar{\Lambda}^{\mu}{}_{\alpha l} + \sigma^{k\mu\dot{\alpha}}\,\Lambda^{\dot{\nu}}{}_{\dot{\alpha}l} = 0 \qquad (2.12a)$$

Indeed, we have

$$\sigma^{k\mu\dot{\nu}}{}_{///l} = \sigma^{k\mu\dot{\nu}}{}_{,l} + \sigma^{r\mu\dot{\nu}}\,\Delta^{k}{}_{rl} = 0 \qquad (2.12b)$$

where $\Delta^{k}{}_{rl}$ is the *Einstein* affinity (1.17)

$$\Delta^{k}{}_{rl} = h_{A}{}^{k}\,h^{A}{}_{r,l} = \sigma^{k\mu\dot{\nu}}\,\sigma_{r\mu\dot{\nu},l} \qquad (2.13)$$

(From (2.10) and (2.4) it follows

$$\gamma_{\alpha\beta////l} = \gamma_{\alpha\beta//l} = -\gamma_{\alpha\lambda}\,\Lambda^{\lambda}{}_{\beta l} - \gamma_{\lambda\beta}\,\Lambda^{\lambda}{}_{\alpha l}$$
$$= -\Lambda_{\alpha\beta l} + \Lambda_{\beta\alpha l} = 0 \qquad (2.14)$$

There exists a duality between the coordinate covariant equations of tensor fields and the *Lorentz* covariant equations for the corresponding spinor field of even degree:

$$\phi_{\mu\dot{\nu}//l} = (\sigma_{i\mu\dot{\nu}}\,\phi^{i})_{///l} = \sigma_{i\mu\dot{\nu}}\,\phi^{i}{}_{;l} \qquad (2.15)$$

With (2.2) and (1.14) and with

$$\phi_{\mu\dot{\nu}} = \phi^{k}\,\sigma_{k\mu\dot{\nu}} = \phi^{A}\,\sigma_{A\mu\dot{\nu}}$$

we also get

$$\phi_{\mu\dot{\nu}///l}\,\phi^{A}{}_{//l}\,\sigma_{A\mu\dot{\nu}} + \phi^{A}\,\sigma_{A\mu\dot{\nu}//l} \qquad (2.16)$$

But because of (2.12) and (1.16) we have

$$= (\sigma^{i\mu\dot{\nu}}\,h^{A}{}_{i})_{///l}$$
$$\sigma^{A}{}_{\mu\dot{\nu}///l} = \sigma^{i\mu\dot{\nu}}{}_{///l}\,h^{A}{}_{i} + \sigma^{i\mu\dot{\nu}}\,h^{A}{}_{i///l} = 0 \qquad (2.17)$$

For this it follows from (2.15) and (2.10):

$$\phi_{\mu\dot{\nu}//l}\,\sigma^{A\mu\dot{\nu}} = \phi^{A}{}_{//l} = h^{A}{}_{i}\,\phi^{i}{}_{;l} \qquad (2.18)$$

For spinors of even degree dual to tensor fields the spinor derivative and the *Lorentz* covariant derivative are equivalent. The *Lorentz* covariant equations for spinor fields of odd degree cannot be formed into coordinate covariant equations without spinor indices.

As it has to be from reasons deriving from the theory of cognition, the equations for any spinor fields can be formulated coordinate invariant and—corresponding to the principle of relativity—also *Lorentz* covariant. But for spinors of odd degree there is no representation which is *Lorentz* invariant and coordinate covariant. Accordingly, the general version of the principle of general relativity is represented by the postulate that all equation of physics can be formulated in Lorentz covariant and coordinate invariant terms. For tensorial quantities we have then also the dual version, that the equations of physics, concerning tensors, can be formulated in *Lorentz* invariant and coordinate covariant terms.

By the coordinate invariant and *Lorentz* covariant formulation of the physical equations, the principle of general relativity can be realised for spinor as for tensor fields; a break-down of this principle in the sense of the general *Lorentz* covariance of the relations between physical quantities can take place only in this way: the geometry of the space-time V_4 is determined not only by the *Lorentz* invariant metric g_{ik}, but also by some non-invariant combinations of the tetrad field $h^A{}_i$. That implies, the laws determining the structure of the space-time have to be continually coordinate covariant. If these equations determine only the metric g_{ik}, then the structure of the space is *Lorentz* invariant. If also some non-invariant combinations of the $h^A{}_i$ are determined, then the general *Lorentz* covariance of the geometric structure is broken off. Especially, the general *Lorentz* covariance of the structure of the V_4 is repealed in full, if all sixteen components of the tetrad field $h^A{}_i$ are determined by the structure equations of the space-time V_4 (Treder, 1967).

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